2-4 Writing Linear Equations

Main Ideas

- Write an equation of a line given the slope and a point on the line.
- Write an equation of a line parallel or perpendicular to a given line.

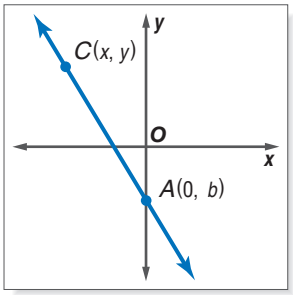
New Vocabulary

slope-intercept form
point-slope form

GET READY for the Lesson

When a company manufactures a product, they must consider two types of cost. There is the *fixed cost*, which they must pay no matter how many of the product they produce, and there is *variable cost*, which depends on how many of the product they produce. In some cases, the total cost can be found using a linear equation such as $y = 5400 + 1.37x$.

Forms of Equations Consider the graph at the right. The line passes through $A(0, b)$ and $C(x, y)$. Notice that b is the y-intercept of $\overleftrightarrow{AC}$. You can use these two points to find the slope of $\overleftrightarrow{AC}$. Substitute the coordinates of points A and C into the slope formula.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$m = \frac{y - b}{x - 0}$$ $(x_1, y_1) = (0, b), (x_2, y_2) = (x, y)$

$$m = \frac{y - b}{x}$$ Simplify.

Now solve the equation for y.

$$mx = y - b$$ Multiply each side by x.

$$mx + b = y$$ Add b to each side.

$$y = mx + b$$ Symmetric Property of Equality

When an equation is written in this form, it is in **slope-intercept form**.

Study Tip

Slope-Intercept Form

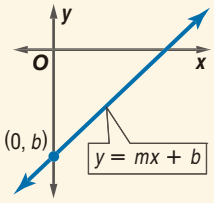
The equation of a vertical line cannot be written in slope-intercept form because its slope is undefined.

KEY CONCEPT *Slope-Intercept Form of a Linear Equation*

Words The slope-intercept form of the equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept.

Symbols $y = mx + b$ (slope: m; y-intercept: b)

Model



If you are given the slope and y-intercept of a line, you can find an equation of the line by substituting the values of m and b into the slope-intercept form. You can also use the slope-intercept form to find an equation of a line if you know the slope and the coordinates of any point on the line.

EXAMPLE Write an Equation Given Slope and a Point

Concepts in Motion
Interactive Lab
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1 **Write an equation in slope-intercept form for the line that has a slope of $-\frac{3}{2}$ and passes through $(-4, 1)$.**

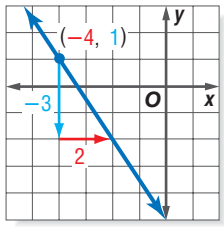
$y = mx + b$ Slope-intercept form

$1 = -\frac{3}{2}(-4) + b$ $(x, y) = (-4, 1), m = -\frac{3}{2}$

$1 = 6 + b$ Simplify.

$-5 = b$ Subtract 6 from each side.

The equation in slope-intercept form is $y = -\frac{3}{2}x - 5$.



CHECK Your Progress

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

1A. slope $\frac{4}{3}$, passes through $(3, 2)$ **1B.** slope -4, passes through $(-2, -2)$

If you are given the coordinates of two points on a line, you can use the **point-slope form** to find an equation of the line that passes through them.

KEY CONCEPT Point-Slope Form of a Linear Equation

Words The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line.

Symbols slope

$$y - y_1 = m(x - x_1)$$

coordinates of point on line

STANDARDIZED TEST EXAMPLE Write an Equation Given Two Points

2 What is an equation of the line through $(-1, 4)$ and $(-4, 5)$?

A $y = -\frac{1}{3}x + \frac{11}{3}$ **B** $y = \frac{1}{3}x + \frac{13}{3}$ **C** $y = -\frac{1}{3}x + \frac{13}{3}$ **D** $y = -3x + 1$

Read the Test Item

You are given the coordinates of two points on the line.

Solve the Test Item

First, find the slope of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{5 - 4}{-4 - (-1)}$ $(x_1, y_1) = (-1, 4)$, $(x_2, y_2) = (-4, 5)$

$= \frac{1}{-3}$ or $-\frac{1}{3}$ Simplify.

Then write an equation.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 4 = -\frac{1}{3}[x - (-1)]$ $m = -\frac{1}{3}$; use either point for (x_1, y_1).

$y = -\frac{1}{3}x + \frac{11}{3}$ The answer is A.

Test-Taking Tip

To check your answer, substitute each ordered pair into your answer. Each should satisfy the equation.

CHECK Your Progress

2. What is an equation of the line through $(2, 3)$ and $(-4, -5)$?

F $y = \frac{4}{3}x + \frac{1}{3}$ **G** $y = \frac{4}{3x} + 8$ **H** $y = \frac{1}{3}x + \frac{17}{3}$ **J** $y = \frac{1}{3}x - 8$

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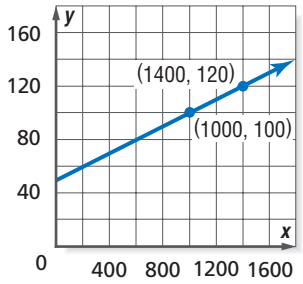
When changes in real-world situations occur at a linear rate, a linear equation can be used as a model for describing the situation.

Real-World EXAMPLE

3 **SALES** **As a salesperson, Eric Fu is paid a daily salary plus commission. When his sales are $1000, he makes $100. When his sales are $1400, he makes $120.**

a. Write a linear equation to model this situation.

Let x be his sales and let y be the amount of money he makes. Use the points (1000, 100) and (1400, 120) to make a graph to represent the situation.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{120 - 100}{1400 - 1000}$$ $(x_1, y_1) = (1000, 100)$, $(x_2, y_2) = (1400, 120)$

$$= 0.05$$ Simplify.

Now use the slope and either of the given points with the point-slope form to write the equation.

$$y - y_1 = m(x - x_1)$$ Point-slope form

$$y - 100 = 0.05(x - 1000)$$ $m = 0.05$, $(x_1, y_1) = (1000, 100)$

$$y - 100 = 0.05x - 50$$ Distributive Property

$$y = 0.05x + 50$$ Add 100 to each side.

The slope-intercept form of the equation is $y = 0.05x + 50$.

b. What are Mr. Fu's daily salary and commission rate?

The y-intercept of the line is 50. The y-intercept represents the money Eric would make if he had no sales. In other words, $50 is his daily salary.

The slope of the line is 0.05. Since the slope is the coefficient of x, which is his sales, he makes 5% commission.

c. How much would Mr. Fu make in a day if his sales were $2000?

Find the value of y when $x = 2000$.

$$y = 0.05x + 50$$ Use the equation you found in part **a**.

$$= 0.05(2000) + 50$$ Replace x with 2000.

$$= 100 + 50 \text{ or } 150$$ Simplify.

Mr. Fu would make $150 if his sales were $2000.

Study Tip

Alternative Method

You could also find Mr. Fu's salary in part c by extending the graph. Then find the y-value when x is 2000.

CHECK Your Progress

SCHOOL CLUBS **For each meeting of the Putnam High School book club, $25 is taken from the activities account to buy snacks and materials. After their sixth meeting, there will be $350 left in the activities account.**

3A. If no money is put back into the account, what equation can be used to show how much money is left in the activities account after having x number of meetings?

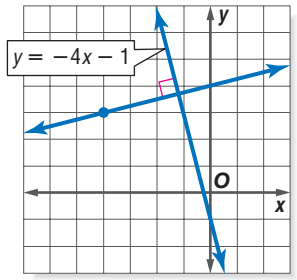
3B. How much money was originally in the account?

3C. After how many meetings will there be no money left in the activities account?

Parallel and Perpendicular Lines The slope-intercept and point-slope forms can be used to find equations of lines that are parallel or perpendicular to given lines.

EXAMPLE 4 Write an Equation of a Perpendicular Line

Write an equation for the line that passes through (−4, 3) and is perpendicular to the line whose equation is $y = -4x - 1$.



The slope of the given line is −4. Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular line is $\frac{1}{4}$.

Use the point-slope form and the ordered pair (−4, 3).

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 3 = \frac{1}{4}[x - (-4)]$ $(x_1, y_1) = (-4, 3), m = \frac{1}{4}$

$y - 3 = \frac{1}{4}x + 1$ Distributive Property

$y = \frac{1}{4}x + 4$ Add 3 to each side.

CHECK Your Progress

4. Write an equation for the line that passes through (3, 7) and is perpendicular to the line whose equation is $y = \frac{3}{4}x - 5$.

CHECK Your Understanding

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

Example 1 (p. 80)

1. slope 0.5, passes through (6, 4)

2. slope $-\frac{3}{4}$, passes through $\left(2, \frac{1}{2}\right)$

3. slope 3, passes through (0, −6)

4. slope 0.25, passes through (0, 4)

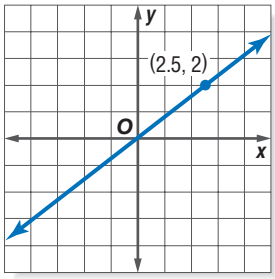
Example 2 (p. 80)

5. passes through (6, 1) and (8, −4)

6. passes through (−3, 5) and (2, 2)

Write an equation in slope-intercept form for each graph.

7.



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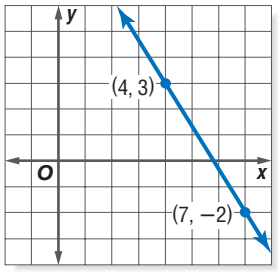


9. STANDARDIZED TEST PRACTICE What is an equation of the line through (2, −4) and (−3, −1)?

A $y = -\frac{3}{5}x + \frac{26}{5}$

B $y = -\frac{3}{5}x - \frac{14}{5}$

C $y = \frac{3}{5}x - \frac{26}{5}$

D $y = \frac{3}{5}x + \frac{14}{5}$

Example 3 (p. 81)

10. PART-TIME JOB Each week Carmen earns $15 plus $0.17 for every pamphlet that she delivers. Write an equation that can be used to find how much Carmen earns each week. How much will she earn the week she delivers 300 pamphlets?

Example 4 (p. 82)

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

11. perpendicular to $y = \frac{3}{4}x - 2$, passes through (2, 0)

12. perpendicular to $y = \frac{1}{2}x + 6$, passes through (−5, 7)

Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
13–16	1
17, 18, 21, 22	2
19, 20	4
23, 24	3

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

13. slope 3, passes through (0, −6)

14. slope 0.25, passes through (0, 4)

15. slope $-\frac{1}{2}$, passes through (1, 3)

16. slope $\frac{3}{2}$ passes through (−5, 1)

17. passes through (−2, 5) and (3, 1)

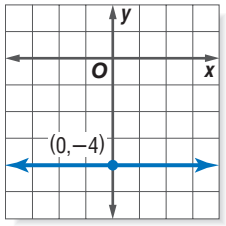
18. passes through (7, 1) and (7, 8)

19. passes through (4, 6), parallel to the graph of $y = \frac{2}{3}x + 5$

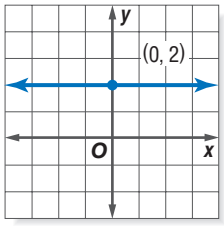
20. passes through (2, −5), perpendicular to the graph of $y = \frac{1}{4}x + 7$

Write an equation in slope-intercept form for each graph.

21.



22.



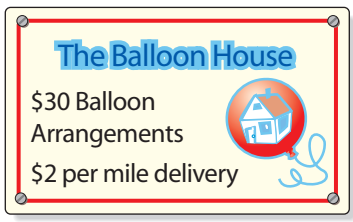
Real-World Link

The number of whitetail deer in the United States increased from about half a million in the early 1900s to 25 to 30 million in 2005.

Source: espn.com

23. ECOLOGY A park ranger at Creekside Woods estimates there are 6000 deer in the park. She also estimates that the population will increase by 75 deer each year to come. Write an equation that represents how many deer will be in the park in x years.

24. BUSINESS For what distance do the two stores charge the same amount for a balloon arrangement?



GEOMETRY For Exercises 25–27, use the equation $d = 180(c - 2)$ that gives the total number of degrees d in any convex polygon with c sides.

25. Write this equation in slope-intercept form.

26. Identify the slope and d-intercept.

27. Find the number of degrees in a pentagon.

SCIENCE For Exercises 28–30, use the following information.

Ice forms at a temperature of 0°C, which corresponds to a temperature of 32°F. A temperature of 100°C corresponds to a temperature of 212°F.

28. Write and graph the linear equation that gives the number y of degrees Fahrenheit in terms of the number x of degrees Celsius.

29. What temperature corresponds to 20°C?

30. What temperature is the same on both scales?

Cliff Keeler/Alamy Images

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

31. slope -0.5, passes through $(2, -3)$

32. slope 4, passes through the origin

33. x-intercept -4, y-intercept 4

34. x-intercept $\frac{1}{3}$, y-intercept $-\frac{1}{4}$

35. passes through $(6, -5)$, perpendicular to the line whose equation is $3x - \frac{1}{5}y = 3$

36. passes through $(-3, -1)$, parallel to the line that passes through $(3, 3)$ and $(0, 6)$

EXTRA PRACTICE
See pages 894, 927.
MathOnline
Self-Check Quiz at algebra2.com

H.O.T. Problems

37. OPEN ENDED Write an equation of a line in slope-intercept form.

38. REASONING What are the slope and y-intercept of the equation $cx + y = d$?

39. CHALLENGE Given $\triangle ABC$ with vertices $A(-6, -8)$, $B(6, 4)$, and $C(-6, 10)$, write an equation of the line containing the altitude from A. (*Hint:* The altitude from A is a segment that is perpendicular to $\overline{BC}$.)

40. *Writing in Math* Use the information on page 79 to explain how linear equations apply to business. Relate the terms *fixed cost* and *variable cost* to the equation $y = 5400 + 1.37x$, where y is the cost to produce x units of a product. Give the cost to produce 1000 units of the product.

STANDARDIZED TEST PRACTICE

41. ACT/SAT What is an equation of the line through $\left(\frac{1}{2}, -\frac{3}{2}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$?

A $y = -2x - \frac{1}{2}$

B $y = -3x$

C $y = 2x - \frac{5}{2}$

D $y = \frac{1}{2}x + 1$

42. REVIEW The total cost c in dollars to go to a fair and ride n roller coasters is given by the equation

$$c = 15 + 3n.$$

If the total cost was \$33, how many roller coasters were ridden?

F 6

G 7

H 8

J 9

Spiral Review

Find the slope of the line that passes through each pair of points. (Lesson 2-3)

43. $(7, 2), (5, 6)$

44. $(1, -3), (3, 3)$

45. $(-5, 0), (4, 0)$

46. INTERNET A Webmaster estimates that the time (seconds) to connect to the server when n people are connecting is given by $t(n) = 0.005n + 0.3$. Estimate the time to connect when 50 people are connecting. (Lesson 2-2)

Solve each inequality. (Lessons 1-5 and 1-6)

47. $|x - 2| \leq -99$

48. $-4x + 7 \leq 31$

49. $2(r - 4) + 5 \geq 9$

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the median of each set of numbers.** (Page 760)

50. $\{3, 2, 1, 3, 4, 8, 4\}$

51. $\{9, 3, 7, 5, 6, 3, 7, 9\}$

52. $\{138, 235, 976, 230, 412, 466\}$

53. $\{2.5, 7.8, 5.5, 2.3, 6.2, 7.8\}$